Problem 4 : Unexpected Blowups

Euler

Using the Euler discretization we have

$$y_{n+1} = y_n + h \cdot (ky_n) = (1 + hk)y_n.$$

Hence, we have $y_n = (1 + hk)^n y_0 = (1 + hk)$. This converges exactly on the set

$$\{h \in \mathbb{R}{\geq}\ 0\ :\ |1 + hk| < 1\}.$$

So to determine if $\lim_{n\to\infty} y_n = 0$, we calculate $|1 + hk|$ and see if it is smaller than 1.

Trapezoid

Using the Trapezoid discretization we have

$$y_{n+1} = y_n + \frac{1}{2}h(ky_n + ky_{n+1})$$

Solving for y_{n+1} gives

$$y_{n+1} = \frac{1 + \frac{1}{2}hk}{1 - \frac{1}{2}hk} y_n.$$

This converges exactly on the set

$$\{h \in \mathbb{R}{\geq}\ 0\ :\ \left|\frac{1 + \frac{1}{2}hk}{1 - \frac{1}{2}hk}\right| < 1\}.$$

So to determine if $\lim_{n\to\infty} y_n = 0$, we calculate $|\frac{1+\frac{1}{2}hk}{1-\frac{1}{2}hk}|$ and see if it is smaller than 1.